**Abstract – For legged locomotion, dynamic balance is one of the most important things to consider. In this paper we considered a foot placement strategy for dynamic balance by modeling the system as bipedal linear inverted pendulum model which is based of the linear inverted pendulum model introduced by Kajita et al. Unlike the linear inverted pendulum model, our model included a double support and single support phases. The results of the simulation showed the same force behavior as it would be expected from a linear inverted pendulum.**

## I. Introduction

Gait analysis and dynamic balance are two topics that are very important for legged locomotion. Gait analysis depends on the type of the system. We could do classification of systems based of number of legs or length of legs. These types of classifications help in determining how the gait analysis would be carried out. For instance, locomotion to systems that have long leg length, such as humans, are driven by gravity that is because the effect of gravity is much bigger than the effect of friction. On the other hand, locomotion for very short leg length, such as mosquitos and bugs, are driven by friction because the effect of friction dominates that of gravity. The system under consideration, in this paper, has a maximum leg length of 1 m so gravity would be the main driver that would be considered for the gait analysis.

For a dynamic system balance is the other thing that should considered. There are several proposed methods for dynamic balance in legged locomotion which include, foot placement, ankle strategy, and hip strategy. One of the simplest theoretical frameworks for walking, which is the type of gait considered for this paper, is the linear inverted pendulum model, LIPM. This framework, which was introduced by Kajita et al. [1], does a very good job accounting for most parameters in walking. LIPM models the body as a point mass with massless legs. There are two main properties of this model. The first is that the center of gravity has no velocity in the vertical direction. The second property is that the motion of the center of gravity could be described by an ordinary linear differential equation. Despite the fact that the LIPM is a very good simplified model, it only considers single support and ignores double support. For this reason, a new method has been introduced by Gayer and Parietti at the 2011 IEEE international conference on robotics and automation. This new model uses the same strategies as LIPM for foot placement but includes double support phase in the gait analysis.

The main objective of this project was to create a simulation based of the bipedal linear inverted pendulum model (bLIP) and compare the results with the model’s prediction.

One of the main motivations for this project is that this theoretical framework was constructed only for 2D case. However, real systems are three dimensional and to be able to test this theory in experiments we would need to expand the theoretical framework into 3D. In order to get a good theoretical model in 3D, however, studying and understanding the bLIP model and verifying the results would be the logical starting point.

## II. Methods

The controller was divided into four main blocks (i.e a stance phase block and a swing phase block for each leg). During the single support when one of the legs is in stance phase, the other leg becomes in swing phase. The stance block in this case takes in Xnow and from that it outputs Fx and Fy. The force in the X is given by equation 1. Since this is a bipedal linear inverted pendulum model, the leg force in the y (vertical direction) should be mg.

$$F_x = \frac{m*g}{yo}*Xnow \quad (1)$$

Where m – mass (which was 50kg for this simulation)

g – Gravitational acceleration (9.81 m/sec$^2$)

y0 – initial height (0.8m)

Xnow – current position of center of mass

The swing phase on the other hand takes $V_{now}$, $X_{now}$, and $\varphi_o$ and outputs where the leg should step at the end of the swing. Since the legs are massless and do not affect the dynamics of the swing, we modeled the swing of the leg about the hip as a damped oscillator [2] which is given by equation 2.

$$J\ddot{\varphi} + r\dot{\varphi} + k\varphi = \varphi_{ref} \quad (2)$$

Where φ is the swing leg angle with respect to the vertical, $\varphi_{ref} = \tan^{-1}\frac{X_T}{y0}$, J is the assumed moment of inertia, k was rotational stiffness, $X_T$ is the target position (as shown in Figure 1 which was taken from Geyer &Parietti) and b was the damping constant. The values of these constants were chosen so that it mimics human leg swing dynamics. Table 1 shows the value of each constant.

Table 1: values of constants

| J | k | b | r | $\omega_0$ |
|---|---|---|---|---|
| 2.4 kgm$^2$ | 726 Nm/rad | 0.5 | = 2J b $\omega_0$ | $\sqrt{\frac{k}{J}}$ |

The simulation was designed so that it reaches the target velocity v* by the end of double support. Therefore, to achieve this target speed the system has to have a change in energy given by equation 3 between now and the end of double support.

$$\Delta E_T = \frac{1}{2}m(\pm v^{*2} - v^2) \quad (3)$$

The ± in front of the target speed indicates whether the target speed and the speed now are in the same direction or in different direction.

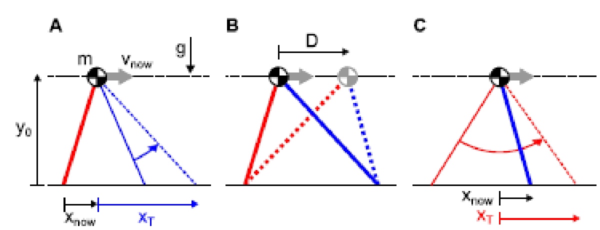


Fig. 1. Bipedal linear inverted pendulum model (bLIPM). (**A**) In single support, one leg is in stance (thick red) while the other (thin blue) swings to a commanded target $x_T$ (dotted blue). (**B**) Once the swing leg lands, the model stays in double support until the commanded length $D$ of the remaining double support goes to zero. (**C**) At that instant, the hind leg takes off and the model reenters single support. *Additional parameters:* $m = 50kg$: mass, $g = 9.81ms^{-2}$: gravitational acceleration, $y_0 = 0.8m$: COM height, $v_{now}$: current speed.

[2]

The target position, $X_T$ was computed with a quadratic equation (equation 5), which was found by equating equation 3 and equation 4, which is the change in energy in double support.

$$\Delta E_{DS}(x_{now}, x_T, D) = \int_{x_{now}}^{x_{now}+D} F_x dx = \frac{mg}{y_0} \frac{D(x_{max} - x_{now} - x_T)(D + x_{now} - x_T)}{2x_{max} - x_{now} - x_T} \quad (4)$$

$$X_T(\Delta E_T) = \frac{-b_1 - \sqrt{b_1{}^2 - 4a_1c_1}}{2a_1} \quad (5)$$

Where, $a_1 = mg(x_{max} - x_{now})$, $b_1 = mg(x_{max} - x_{now})(-2x_{max} + x_{now}) + \Delta E_T y_0$, and $c_1 = mgx_{max}(x_{max} - x_{now})^2 - \Delta E_T y_0(2x_{max} - x_{now})$

Depending on the values of required change in energy, the target position, $X_T$ has the following different values.

$$x_T = \begin{cases} 0, & \Delta E_T > \Delta E_{DS}^{max} \\ x_T(\Delta E_T), & \Delta E_{DS}^{min} \leq \Delta E_T \leq \Delta E_{DS}^{max} \\ x_{T,E_{DS}^{min}}, & \Delta E_T < \Delta E_{DS}^{min} \end{cases} \quad (6)$$

The swing phase switches off when

$|\varphi - \varphi_{ref}| < 1$ degree.

In double support, the swing phase blocks for both legs are off. The stance phase in this case calculates the leg force distribution according to the current geometry. The force in the y direction decreases from mg to zero linearly for the hind leg and increases linearly from zero to mg for the front leg. On the other hand the horizontal forces are calculated by equations 7 and 8 for the hind front leg respectively.

$$F_x^h = mg \frac{x_{max} - x_{now}}{L_{tot}} \frac{x_{now}}{y_0} \quad (7)$$

$$F_x^f = mg\left(1 - \frac{x_{max} - x_{now}}{L_{tot}}\right) \frac{x_{now} - L_{step}}{y_0}$$

Where $L_{tot} = 2x_{max} - L_{step}$ and $x_{max} = \sqrt{l_{max}{}^2 - y_0}$

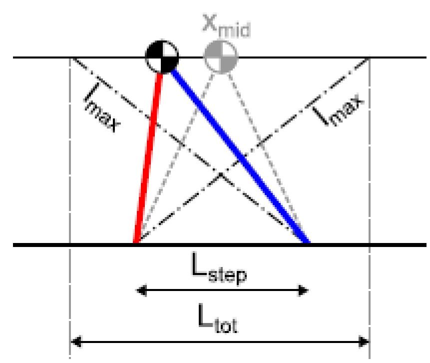


Figure 2: geometry definitions [2]

If the stance leg length gets to $l_{max}$ before the swing leg meets its switching criterion, we would have a case where both legs become in swing phase. Since bLIPM does not model flight, this case implies the model failed to balance [2].

## III. Results

For the parameters given in section II the simulation runs smoothly alternating between single support and double support phases. Some of the results of the simulation are below.

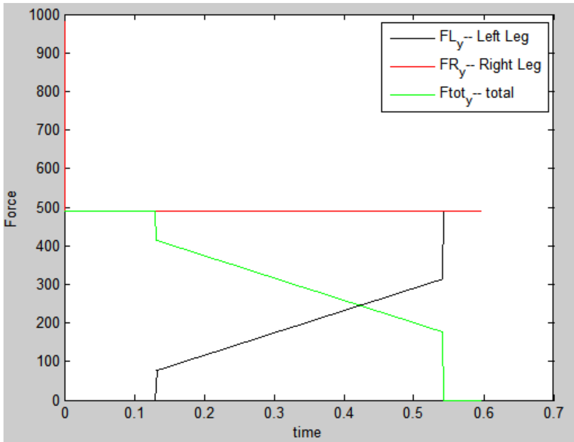


Figure 3: Vertical leg forces (in SI units)

As shown in Figure 3 the total force in the direction is constant with magnitude equal to mg. The vertical force for double support phase (between t = 0.13s and t = 0.54s for each leg is behaving linearly as shown in the green and blue lines in Figure 3.

Figure 4 below shows the horizontal leg forces for each leg as well as the total horizontal leg force for one cycle.

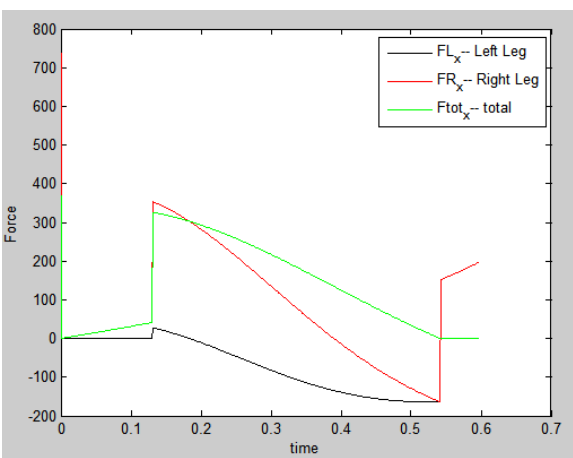


Figure 4: Horizontal leg forces (in SI units)

Note that all the simulation was done without including a collision force. In the case where a collision force of was considered with a gain of -500, the system changed gait into standing instead of walking

## IV. Discussion and Conclusion

The main aim of this project was to be able to simulate bipedal walking using a bipedal inverted pendulum model (bLIPM) as described by Geyer and Perietti [2]. The bLIP model is based on the linear inverted pendulum model (LIPM) which was introduced by Kajita et al. and it predicts a constant vertical force and linear horizontal forces on the legs. Figure 3 and Figure 4 shown in the section III show this expected features. In addition to the leg forces behaving in the predicted manner, the simulation runs very smoothly. This verifies the presumption that this model works well for a 2D case. Therefore, using this model for developing a bipedal linear inverted pendulum model in 3D would be an ideal transition.

One thing that was not fully explored in this model is external disturbances. Therefore, an area that should be looked at in future work would be how this controller acts in different disturbance situations. Also as it was already mentioned several times, this model is only for 2D so developing a 3D theoretical framework should also be considered in future works.

## V. Acknowledgement

I would like to thank Professor Geyer Hartmut as he played a huge role by helping during debugging of the controller as well as theoretical checkups. The matlab simulation codes

were also written and modified based on his original codes.

## References

[1] S. Kajita, K. Tani, and A. Kobayashi, “Dynamic Walk Control of a Biped Robot along the Potential Energy Conserving Orbit,” *Int. workshop on Intelligent Robots and systems IROS ’90* pp 789-794

[2] F. Parietti and G. Hartmut, “Reactive Balance Control in Walking based on a Bipedal Linear Inverted Pendulum Model,” in proc. IEEE *Int. Conf. on Robotics and Automation* 2011 pp 5442-5447